Thermal Noise Sources

The noise power generated by a thermal noise source will be studied since the receiver noise is evaluated in terms of this phenomenon. A conductive element with two terminals may be characterized by its resistance, R ohms. This resistive or lossy element contains free electrons that have some random motion if the resistor has a temperature above absolute zero. This random motion causes a noise voltage to be generated at the terminals of the resistor. Although the noise is small, when the noise is amplified by a high-gain receiver it can become a problem. (If no noise were present, we could communicate to the edge of the universe with infinitely small power since the signal could always be amplified without having noise introduced.)

This physical lossy element, or physical resistor, can be modeled by an equivalent circuit that consists of a noiseless resistor in series with a noise voltage source (see Fig. 5-35). From quantum mechanics, it can be shown that the (normalized) power spectrum corresponding to the voltage source is [Van der Ziel, 1986]

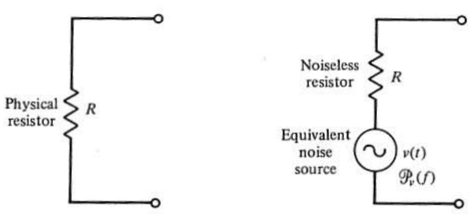
$$\mathcal{P}_v(f) = 2R\left[\frac{h|f|}{2} + \frac{h|f|}{e^{h|f|/(kT)} - 1}\right] \tag{5-59}$$

where R = value of the physical resistor (ohms)
$h = 6.2 \times 10^{-34}$ J-sec is Planck's constant
$k = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant, where K is kelvin
$T = (273 + C)$ is the absolute temperature of the resistor (kelvin)

At room temperature for frequencies below 1000 GHz, $[h|f|/(kT)] < 1/5$, so that $e^x = 1 + x$ is a good approximation. Then (5-59) reduces to

$$\mathcal{P}_v(f) = 2RkT \tag{5-60}$$

This equation will be used to develop other formulas in this text since the RF frequencies of interest are usually well below 1000 GHz and we are not dealing with temperatures near absolute zero.



(a) A Lossy Circuit Element Characterized by Its Resistance

(b) Thevenin Equivalent Circuit Model of the Lossy Circuit Element

FIGURE 5-35 Thermal noise source.

If the open-circuit noise voltage that appears across a physical resistor is read by a true rms voltmeter that has a bandwidth of B hertz, then, using (2-67), the reading would be

$$V_{\text{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{2\int_0^B \mathcal{P}_v(f)\,df} = \sqrt{4kTBR} \tag{5-61}$$

Characterization of Noise Sources

Noise sources may be characterized by the maximum amount of noise power or PSD that can be passed to a load.

DEFINITION: The *available noise power* is the *maximum*† actual (i.e., not normalized) power that can be drawn from a source. The *available PSD* is the *maximum* actual (i.e., not normalized) PSD that can be obtained from a source.

For example, the available PSD for a thermal noise source is easily evaluated using Fig. 5-36 and (2-142).

$$\mathcal{P}_a(f) = \frac{\mathcal{P}_v(f)|H(f)|^2}{R} = \tfrac{1}{2}kT \;\left[\text{W/Hz}\right] \tag{5-62}$$

where $H(f) = \frac{1}{2}$ for the resistor divider network. The available power from a thermal source in a bandwidth of B hertz is

$$P_a = \int_{-B}^{B} \mathcal{P}_a(f)\,df = \int_{-B}^{B} \frac{1}{2}kT\,df$$

or

$$P_a = kTB \tag{5-63}$$

This equation indicates that the available noise power from a thermal source does *not* depend on the value of R, even though the open-circuit rms voltage does. The available noise power from a source (it does not have to be a thermal source) can be specified by a number called the noise temperature.

DEFINITION: The *noise temperature* of a source is given by

$$T = \frac{P_a}{kB} \tag{5-64}$$

where P_a is the available power from the source in a bandwidth of B hertz.

In using this definition, it is noted that if the source happens to be of thermal origin, T will be the temperature of the device in kelvins, but if the source is not

†The maximum power or maximum PSD is obtained when $Z_L(f) = Z_s^*(f)$, where $Z_L(f)$ is the load impedance and $Z_s^*(f)$ is the conjugate of the source impedance.

$P_a = kTB$ = in dB $10\log_{10} kT + 10\log_{10} B$ → in dBm
-204 dBW $+ 10\log_{10} B$; -174 dBm $+ 10\log_{10} B$

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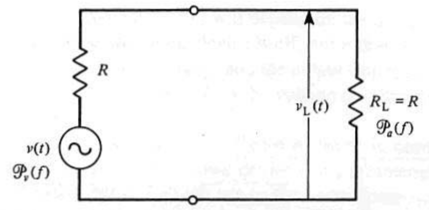

FIGURE 5-36 Thermal source with a matched load.

of thermal origin, the number obtained for T may have nothing to do with the physical temperature of the device.

Noise Characterization of Linear Devices

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A linear device with internal noise generators may be modeled as shown in Fig. 5-37. Any device that can be built will have some internal noise sources. As shown in the figure, the device could be modeled as a noise-free device having a power gain $G_a(f)$ and an excess noise source at the output to account for the internal noise of the actual device. Some examples of linear devices that have to be characterized in receiving systems are lossy transmission lines, RF amplifiers, down converters, and IF amplifiers.

The power gain of the devices is the available power gain.

Definition: The *available power gain* of a linear device is

$$G_a(f) = \frac{\text{available PSD out of the device}}{\text{available PSD out of the source}} = \frac{\mathcal{P}_{ao}(f)}{\mathcal{P}_{as}(f)} \tag{5-65}$$

When $\mathcal{P}_{ao}(f)$ is measured to obtain $G_a(f)$, the source noise power is made large enough so that the amplified *source* noise that appears at the output dominates any other noise. In addition, note that $G_a(f)$ is defined in terms of actual

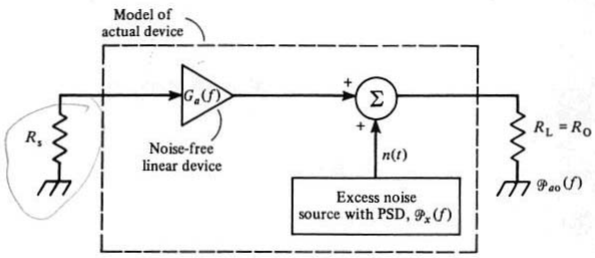

FIGURE 5-37 Noise model for an actual device.

(i.e., not normalized) PSD. In general, $G_a(f)$ will depend on the driving source impedance as well as on elements within the device itself, but it does *not* depend on the load impedance. If the source impedance and the output impedance of the device are equal, $G_a(f) = |H(f)|^2$, where $H(f)$ is the voltage or current transfer function of the linear device.

To characterize the goodness of a device, a figure of merit is needed that compares the actual (noisy) device with an ideal device (i.e., no internal noise sources). Two figures of merit, both of which tell us the same thing—namely, how bad the noise performance of the actual device is—are universally used. They are noise figure and effective input-noise temperature.

K→C °C = K − 273,15   C→F °F = (°C · 9/5) + 32   F→C °C = (°F − 32) · 5/9

Definition: The *spot noise figure* of a linear device is obtained by terminating the device with a thermal noise source of temperature T_0 on the input and a matched load on the output as indicated in Fig. 5-37. The spot noise figure is

$$F_s(f) = \frac{\text{measured available PSD out of the actual device}}{\text{available PSD out of an ideal device with the same available gain}}$$

or

$$F_s(f) = \frac{\mathcal{P}_{ao}(f)}{(kT_0/2)G_a(f)} = \frac{(kT_0/2)G_a(f) + \mathcal{P}_x(f)}{(kT_0/2)G_a(f)} > 1 \qquad (5\text{-}66)$$

The value of R_s is the same as the source resistance that was used when $G_a(f)$ was evaluated. A standard temperature of $T_0 = 290$ K is used as adopted by the IEEE [Haus, 1963].

$F_s(f)$ is called the *spot noise figure* since it refers to the noise characterization at a particular "spot" or frequency in the spectrum. Note that $F_s(f)$ is always greater than unity for an actual device, but it is nearly unity if the device is almost an ideal device. $F_s(f)$ is a function of the source temperature, T_0. Consequently, when the noise figure is evaluated, a standard temperature of $T_0 = 290$ K is used. This corresponds to a room temperature of 62.3°F. (≈17°C)

Often an average noise figure instead of a spot noise figure is desired. The average is measured over some bandwidth B.

Definition: The *average noise figure* is

$$F = \frac{P_{ao}}{kT_0 \int_{f_0-B/2}^{f_0+B/2} G_a(f)\, df} \qquad (5\text{-}67)$$

where

$$P_{ao} = 2\int_{f_0-B/2}^{f_0+B/2} \mathcal{P}_{ao}(f)\, df$$

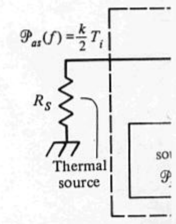

FIGURE 5-3

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Definition: The *spo*
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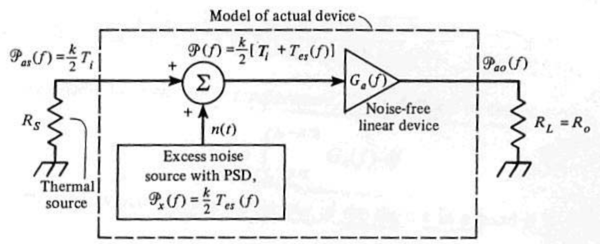

FIGURE 5-38 Another noise model for an actual device.

is the measured available output in a bandwidth B hertz wide centered on a frequency of f_0 and $T_0 = 290$ K.

If the available gain is constant over the band so that $G_a(f) = G_a$ over the frequency interval $(f_0 - B/2) \leq f \leq (f_0 + B/2)$, the noise figure becomes

$$F = \frac{P_{ao}}{kT_0BG_a} \tag{5-68a}$$

The noise figure is often measured by using the Y-factor method. This technique is illustrated by Prob. 5-41. The Hewlett-Packard HP8970B Noise Figure Meter uses the Y-factor method for measuring the noise figure of devices.

The noise figure can also be specified in decibel units,†

$$F_{\text{dB}} = 10 \log(F) = 10 \log\left(\frac{P_{ao}}{kT_0BG_a}\right) \tag{5-68b}$$

For example, suppose that the manufacturer of an RF preamplifier specifies that an RF preamp has a 2-dB noise figure. This means that the actual noise power at the output is 1.58 times the power that would occur because of amplification of noise from the input. The other figure of merit for evaluating the noise performance of a linear device is the effective input-noise temperature, which is illustrated in Fig. 5-38.

DEFINITION: The *spot effective input-noise temperature*, $T_{es}(f)$, of a linear device is the *additional* temperature required for an input source, which is driving an ideal (noise-free) device, to produce the same available PSD at the ideal device output as is obtained from the actual device when it is driven by the input source of temperature T_i kelvin. That is, $T_{es}(f)$ is defined by

$$\mathcal{P}_{ao}(f) = G_a(f)\,\frac{k}{2}\,[T_i + T_{es}(f)] \tag{5-69}$$

†Some authors call F the *noise factor* and F_{dB} the *noise figure*.

where $\mathcal{P}_{ao}(f)$ is the available PSD out of the actual device when driven by an input source of temperature T_i, and $T_{es}(f)$ is the spot effective input-noise temperature.

The *average effective input-noise temperature*, T_e, is defined by the equation

$$P_{ao} = k(T_i + T_e)\int_{f_0-B/2}^{f_0+B/2} G_a(f)\, df \tag{5-70}$$

where the measured available noise power out of the device in a band B hertz wide is

$$P_{ao} = 2\int_{f_0-B/2}^{f_0+B/2} \mathcal{P}_{ao}(f)\, df \tag{5-71}$$

Since $G_a(f)$ depends on the source impedance as well as on the device parameters, $T_e(f)$ will depend on the source impedance used as well as on the characteristics of the device itself, but it is independent of the value of T_i used. In the definition of T_e, note that the IEEE standards do not specify that $T_i = T_0$ since the value of T_e obtained does not depend on the value of T_i that is used. However, $T_i = T_0 = 290$ may be used for convenience. The effective input-noise temperature can also be evaluated by the Y-factor method, as illustrated by Prob. 5-41.

When the gain is flat (constant) over the frequency band, $G_a(f) = G_a$, the effective input-noise temperature is simply

$$T_e = \frac{P_{ao} - kT_iG_aB}{kG_aB} \tag{5-72}$$

Note that $T_{es}(f)$ and T_e are greater than zero for an actual device, but if the device is nearly ideal (small internal noise sources), they will be very close to zero.

When T_e was evaluated by use of (5-70), an input source was used with some convenient value for T_i. However, when the device is used in a system, the available noise power from the source will be different if T_i is different. For example, suppose that the device is an RF preamplifier and the source is an antenna. The available power out of the amplifier when it is connected to the antenna is now†

dalla 5-70 $$P_{ao} = 2\int_{f_0-B/2}^{f_0+B/2} \mathcal{P}_{as}(f)G_a(f)\, df + kT_e\int_{f_0-B/2}^{f_0+B/2} G_a(f)\, df \tag{5-73}$$

where $\mathcal{P}_{as}(f)$ is the available PSD out of the source (antenna). T_e is the average effective input temperature of the amplifier that was evaluated by using the input source T_i. If the gain of the amplifier is approximately constant over the band, this reduces to

$$P_{ao} = G_aP_{as} + kT_eB\, G_a \tag{5-74}$$

†The value of P_{ao} in (5-73) and (5-74) is different from that in (5-70), (5-71), and (5-72).

where the available power from the source (antenna) is

$$P_{as} = 2\int_{f_0-B/2}^{f_0+B/2} \mathcal{P}_{as}(f)\, df \tag{5-75}$$

Furthermore, the available power from the source might be characterized by its noise temperature, T_s, so that, using (5-64),

$$P_{as} = kT_sB \tag{5-76}$$

In satellite Earth station receiving applications, the antenna (source) noise temperature might be $T_s = 32$ K at 4 GHz for a parabolic antenna with a diameter of 5 m, where the noise from the antenna is due to cosmic radiation and to energy received from the ground as the result of the sidelobe beam pattern of the antenna. (The Earth acts as a blackbody noise source with $T = 280$ K.) Note that the $T_s = 32$ K of the antenna is "caused" by *radiation* resistance, which is not the same as a loss resistance (I^2R losses) associated with a thermal source and T_s has no relation to the physical temperature of the antenna.

In summary two figures of merit have been defined: noise figure and effective input-noise temperature. By combining (5-66) and (5-69) where $T_i = T_0$, a relationship between these two figures of merit for the spot measures is obtained:

$$T_{es}(f) = T_0[F_s(f) - 1] \tag{5-77a}$$

Here $T_i = T_0$ is required because $T_i = T_0$ is used in the definition for the noise figure that precedes (5-66).

Using (5-68a) and (5-72) where $T_i = T_0$, we obtain the same relationship for the average measures:

$$T_e = T_0(F - 1) \tag{5-77b}$$

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EXAMPLE 5-4 T_e AND F FOR A TRANSMISSION LINE

The effective input-noise temperature, T_e, and the noise figure, F, for a lossy transmission line (a linear device) will now be evaluated.† This can be accomplished by terminating the transmission line with a source and a load resistance (all having the same physical temperature) that are both equal to the characteristic impedance of the line, as shown in Fig. 5-39. The gain of the transmission line is $G_a = 1/L$, where L is the transmission line loss (power in divided by power out) in absolute units (i.e., not dB units). Looking into the output port of the transmission line, one sees an equivalent circuit that is resistive (thermal source) with a value of R_0 ohms since the input of the line is terminated by a resistor of R_0 ohms (the characteristic impedance). Assume that the physical temperature of the transmission line is T_L, as measured on the Kelvin scale. Since the line acts as a thermal source, the available noise power at its output is

†These results also hold for the T_e and F of (impedance) matched attenuators.

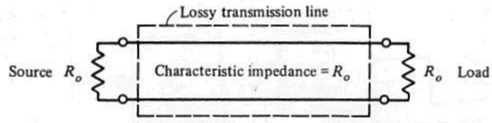

FIGURE 5-39 Noise figure measurement of a lossy transmission line.

$P_{ao} = kT_L B$. Using (5-72) where the source is at the same physical temperature, $T_i = T_L$, we get

$T_i \to T_L$

$$T_e = \frac{kT_L B - kT_i G_a B}{kG_a B} = T_L\left(\frac{1}{G_a} - 1\right)$$

Thus the effective input-noise temperature for the transmission line is

$$T_e = T_L(L - 1) \tag{5-78a}$$

where T_L is the physical temperature (Kelvin) of the line and L is the line loss. If the physical temperature of the line happens to be T_0, this becomes

$$T_e = T_0(L - 1) \tag{5-78b}$$

The noise figure for the transmission line is obtained by using (5-77b) to convert T_e to F. Thus, substituting (5-78a) into (5-77b), we get

$$T_L(L - 1) = T_0(F - 1)$$

Solving for F, we obtain the noise figure for the transmission line

$$F = 1 + \frac{T_L}{T_0}(L - 1) \tag{5-79a}$$

where T_L is the physical temperature (Kelvin) of the line, $T_0 = 290$, and L is the line loss. If the physical temperature of the line is 290 K (63°F), (5-79a) reduces to

$$F = \frac{1}{G_a} = L \tag{5-79b}$$

In decibel measure, this is $F_{\text{dB}} = L_{\text{dB}}$. In other words, if a transmission line has a 3-dB loss, it has a noise figure of 3 dB provided that it has a physical temperature of 63°F. If the temperature is 32°F (273 K), the noise figure, using (5-79a), will be 2.87 dB. Thus F_{dB} is approximately L_{dB}, if the transmission line is located in an environment (temperature range) that is inhabitable by humans.

Noise Characterization of Cascaded Linear Devices

In a communication system several linear devices, supplied by different vendors, are often cascaded together to form an overall system, as indicated in Fig. 5-40. In a receiving system these devices might be an RF preamplifier connected to a

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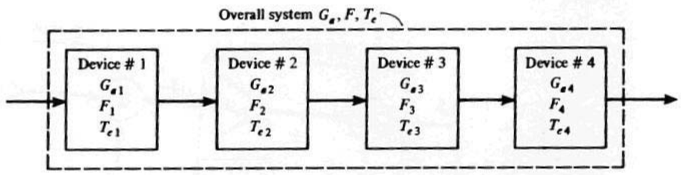

FIGURE 5-40 Cascade of four devices.

transmission line that feeds a down converter and an IF amplifier. (As discussed in Sec. 4-3, the down converter is a linear device and may be characterized by its conversion power gain and its noise figure.) For system performance calculations, we need to evaluate the overall power gain, G_a, and the overall noise characterization (which is given by the overall noise figure or the overall effective input-noise temperature) from the specifications for the individual devices provided by the vendors.

The overall available power gain is

$$G_a(f) = G_{a1}(f)G_{a2}(f)G_{a3}(f)G_{a4}(f)\cdots \tag{5-80}$$

since, for example, for a four-stage system,

$$G_a(f) = \frac{\mathcal{P}_{ao4}}{\mathcal{P}_{as}} = \left(\frac{\mathcal{P}_{ao1}}{\mathcal{P}_{as}}\right)\left(\frac{\mathcal{P}_{ao2}}{\mathcal{P}_{ao1}}\right)\left(\frac{\mathcal{P}_{ao3}}{\mathcal{P}_{ao2}}\right)\left(\frac{\mathcal{P}_{ao4}}{\mathcal{P}_{ao3}}\right)$$

Theorem: *The overall noise figure for cascaded linear devices is*†

$$F = F_1 + \frac{F_2 - 1}{G_{a1}} + \frac{F_3 - 1}{G_{a1}G_{a2}} + \frac{F_4 - 1}{G_{a1}G_{a2}G_{a3}} + \cdots \tag{5-81}$$

as shown in Fig. 5-40 (for a four-stage system).

Proof. This result may be obtained by using an excess-noise model of Fig. 5-37 for each stage. We will prove the result for a two-stage system as modeled in Fig. 5-41. The overall noise figure is

$$F = \frac{P_{ao2}}{(P_{ao2})_{\text{ideal}}} = \frac{P_{x2} + P_{ao1}G_{a2}}{G_{a1}G_{a2}P_{as}}$$

which becomes

$$F = \frac{P_{x2} + G_{a2}(P_{x1} + G_{a1}P_{as})}{G_{a1}G_{a2}P_{as}} \tag{5-82}$$

where $P_{as} = kT_0B$ is the available power from the thermal source. P_{x1} and P_{x2} can be obtained from the noise figures of the individual devices by using Fig.

†This is known as Friis's noise formula.

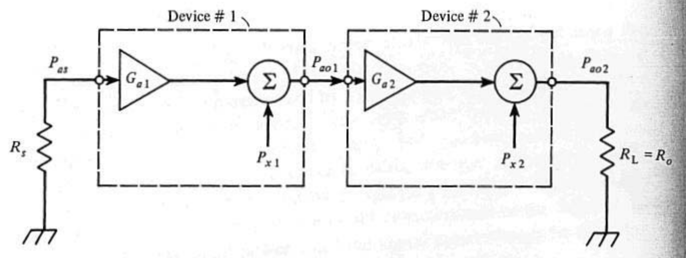

FIGURE 5-41 Noise model for two cascaded devices.

5-37, so that for the ith device

$$F_i = \frac{P_{aoi}}{G_{ai}P_{as}} = \frac{P_{xi} + G_{ai}P_{as}}{G_{ai}P_{as}}$$

or

$$P_{xi} = G_{ai}P_{as}(F_i - 1) \tag{5-83}$$

Substituting this equation into (5-82) for P_{x1} and P_{x2}, we obtain

$$F = F_1 + \frac{F_2 - 1}{G_{a1}}$$

which is identical to (5-81) for the case of two cascaded stages. In a similar way (5-81) can be shown to be true for any number of stages.

Looking at (5-81), we see that if the terms G_{a1}, $G_{a1}G_{a2}$, $G_{a1}G_{a2}G_{a3}$, and so on, are relatively large, F_1 will dominate the overall noise figure. Thus, in receiving system design, it is important that the first stage have a low noise figure and a large available gain so that the noise figure of the overall system will be as small as possible.

The overall effective input-noise temperature of several cascaded stages can also be evaluated.

Theorem: *The overall effective input-noise temperature for cascaded linear devices is*

$$T_e = T_{e1} + \frac{T_{e2}}{G_{a1}} + \frac{T_{e3}}{G_{a1}G_{a2}} + \frac{T_{e4}}{G_{a1}G_{a2}G_{a3}} + \cdots \tag{5-84}$$

as shown in Fig. 5-40.

A proof of this result is left for the reader as an exercise.

For further study concerning the topics of effective input-noise temperature and noise figure, the reader is referred to an authoritative monograph [Mumford and Scheibe, 1968].

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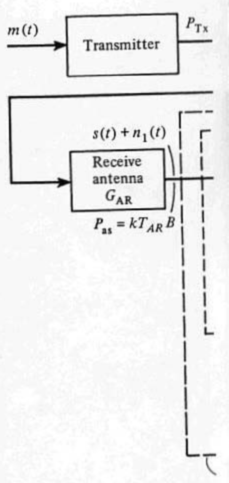

FIGURE 5-42 Con